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# Reteaching with Practice

For use with pages 813–819

Read and do all problems from pages 1 & 2. See page 3 for further instructions.

**GOAL** **Use parametric equations to represent motion in a plane**

**VOCABULARY**

A pair of equations that expresses $x$ and $y$ in terms of a third variable $t$, written as $x = f(t)$ and $y = g(t)$, are called **parametric equations,** and $t$ is called the **parameter.**

## EXAMPLE 1 *Graphing a Set of Parametric Equations*

Graph $x = t - 3$ and $y = -2t + 1$ for $1 \le t \le 5$.

**SOLUTION**

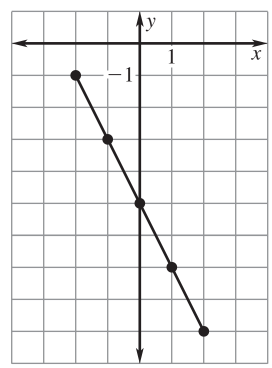
Begin by making a table of values.

| $t$ | 1 | 2 | 3 | 4 | 5 |
|---|---|---|---|---|---|
| $x$ | $-2$ | $-1$ | 0 | 1 | 2 |
| $y$ | $-1$ | $-3$ | $-5$ | $-7$ | $-9$ |

Then plot the points $(x, y)$ given in the table:

$(-2, -1), (-1, -3), (0, -5), (1, -7), (2, -9)$

Now connect the points with a line segment as shown.




### *Exercises for Example 1*

**Graph the parametric equations.**

**1.** $x = 2t + 1$ and $y = -t + 2$ for $0 \le t \le 4$

**2.** $x = t$ and $y = -t$ for $2 \le t \le 6$

**3.** $x = -3t + 4$ and $y = t + 1$ for $1 \le t \le 4$

**4.** $x = -2t + 2$ and $y = t - 1$ for $0 \le t \le 4$

## EXAMPLE 2 *Eliminating the Parameter*

Write an $xy$-equation for the parametric equations in Example 1: $x = t - 3$ and $y = -2t + 1$ for $1 \le t \le 5$. State the domain for the equation.

**SOLUTION**

First solve one of the parametric equations for $t$. It is more convenient to solve the $x$-equation because the coefficient of $t$ is one.

| | |
|---|---|
| $x = t - 3$ | Write original equation. |
| $x + 3 = t$ | Add 3 to each side. |

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Then substitute this value for $t$ in the other parametric equation.

| | |
|---|---|
| $y = -2t + 1$ | Write original equation. |
| $y = -2(x + 3) + 1$ | Substitute for $t$. |
| $y = -2x - 6 + 1$ | Use distributive property. |
| $y = -2x - 5$ | Simplify. |

To find the domain of the $xy$-equation, determine the values of $x$ when $t = 1$ and $t = 5$. When $t = 1$, $x = t - 3 = 1 - 3 = -2$, and when $t = 5$, $x = t - 3 = 5 - 3 = 2$. So, the domain is $-2 \leq x \leq 2$.

### *Exercises for Example 2*

**Write an *xy*-equation for the parametric equations. State the domain.**

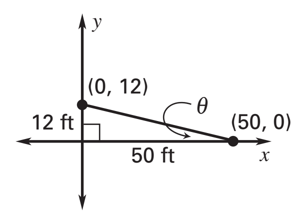
**5.** Exercise 1 **6.** Exercise 2 **7.** Exercise 3 **8.** Exercise 4

## EXAMPLE 3 *Modeling Linear Motion*

An object is at (0, 12) at time $t = 0$ and then at (50, 0) at time 4. Write parametric equations describing the linear motion.

### Solution

The angle of elevation is $\theta = \tan^{-1} \frac{12}{50} \approx 13.5°$. To find the constant speed $V$, divide the distance by the change in time, $t = 4$ seconds.




The distance traveled is the hypotenuse of the right triangle, so

$$d = \sqrt{12^2 + 50^2} = \sqrt{2644} \approx 51.4 \text{ ft and}$$

$$v \approx \frac{51.4 \text{ ft}}{4 \text{ sec}} \approx 12.9 \text{ ft/sec}$$

Using $v = 12.9$, $\theta = 13.5°$, and $(x_0, y_0) = (0, 12)$, you can write:

$$x = (v \cos \theta)t + x_0 \quad \text{and} \quad y = (v \sin \theta)t + y_0$$
$$\approx (12.9 \cos 13.5°)t + 0 \qquad \approx (12.9 \sin 13.5°)t + 12$$
$$\approx 13t \qquad \approx 3t + 12$$

### *Exercises for Example 3*

**Use the given information to write parametric equations describing the linear motion.**

**9.** An object is at (0, 0) at time $t = 0$ and then at (15, 80) at time $t = 5$.

**10.** An object is at (0, 20) at time $t = 0$ and then at (100, 0) at time $t = 10$.

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# Practice C

For use with pages 813–819

Do: 2, 4, 6, 7, 9, 14

**Graph the parametric equations.**

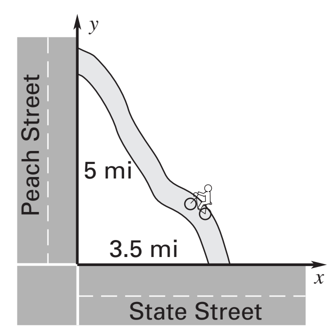
1. $x = 2t$ and $y = 3t + 1$ for $0 \le t \le 4$
2. $x = 2t + 6$ and $y = 4t - 1$ for $2 \le t \le 6$
3. $x = 20t + 10$ and $y = 40t - 20$ for $1 \le t \le 5$
4. $x = (14.1 \cos 30°)t$ and $y = (14.1 \sin 30°)t$ for $0 \le t \le 4$

**Write an *xy*-equation for the parametric equations. State the domain.**

5. $x = t + 7$ and $y = 12 - t$ for $0 \le t \le 5$
6. $x = 2t + 4$ and $y = 6t + 2$ for $0 \le t \le 8$
7. $x = (21.4 \cos 46.1°)t$ and $y = (21.4 \sin 46.1°)t$ for $0 \le t \le 10$
8. $x = (8.1 \cos 65.6°)t$ and $y = (8.1 \sin 65.6°)t$ for $0 \le t \le 15$

**Use the given information to write parametric equations describing the linear motion.**

9. An object is at $(0, 0)$ at time $t = 0$ and then at $(31, 82)$ at time $t = 3$.
10. An object is at $(4, 1)$ at time $t = 0$ and then at $(27, 7)$ at time $t = 5$.
11. An object is at $(-2, 0)$ at time $t = 1$ and then at $(14, 7)$ at time $t = 8$.
12. An object is at $(2, -3)$ at time $t = 5$ and then at $(56, 42)$ at time $t = 12$.
13. ***Soccer*** You are a goalie in a soccer game. You save the ball and then drop kick it as far as you can down the field. Your kick has an initial speed of 30 feet per second and starts at a height of 2.5 feet. If you kick the ball at an angle of 50°, how far down the field does the ball hit the ground?
14. ***Bike Path*** A bike trail connects State and Peach Streets as shown. You enter the trail 3.5 miles from the intersection of the streets and pedal at a speed of 12 miles per hour. You reach Peach Street 5 miles from the intersection. Write a set of parametric equations to describe your path.



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# *Interdisciplinary Applications*

**For use with pages 813–819**

Groups are alphabetical:

Group 1: Do 1 & 4

Group 2: Do 2 & 5

Group 3: Do 3 & 6

Group 4: Do 1 & 5

Group 5: Do 2 & 6

Do each by hand on full sheet of graph paper. Each person in group should do the work and the graph.

## Math in the Toy Store

**TRIGONOMETRY** It is interesting to note how some mathematical ideas find their way into the marketplace. In jewelry stores, for example, two friends can buy necklaces that appear as a broken circle—one with the number "220" on it, the other with the number "284." These numbers were chosen because they are "friendly numbers." Two numbers are "friendly" if each is the sum of the proper divisors of the other. The proper divisors of 220 are 1, 2, 3, 5, 10, 11, 20, 22, 44, 55 and 110, which sum to 284. In the same way, the divisors of 284 sum to 220.

Another example is *Spirograph*™. Spirograph is a toy that had several gear-like shapes (mostly circles) that allows the creation of many fancy shapes and curves by allowing shapes to rotate around other shapes.

The process that creates curves on a *Spirograph*™ — circles rotating on other paths—is perfectly modeled by parametric equations and has been studied by some of the greatest mathematicians of all time.

**Graph the following parametric equations to reveal some of these intricate and interesting shapes.**

**1.** The *Spiral of Archimedes*: $\begin{matrix} x = t\cos t \\ y = t\sin t \end{matrix}$, $0 \le t \le 2\pi$

**2.** The *Cissoid of Diocles*: $\begin{matrix} x = \sin t\cos t\tan t \\ y = \sin^2 t\tan t \end{matrix}$, $0 \le t \le 2\pi$

**3.** The *Ouija Board* curve: $\begin{matrix} x = \dfrac{\sin t\cos t}{t} \\ y = \dfrac{\sin^2 t}{t} \end{matrix}$, $-2\pi \le t \le 2\pi$

**4.** The *Cycloid*: $\begin{matrix} x = t - \sin t \\ y = 1 - \cos t \end{matrix}$, $0 \le t \le 4\pi$

**5.** The *Folium of Descartes*: $\begin{matrix} x = \dfrac{3t}{1 + t^3} \\ y = \dfrac{3t^2}{1 + t^3} \end{matrix}$, $-4\pi \le t \le 4\pi$

**6.** The *Witch of Agnesi*: $\begin{matrix} x = \cot t \\ y = \sin^2 t \end{matrix}$, $0 \le t \le 2\pi$