1.5 Continued…
Logarithms

Base *a* Logarithm Function

- If *a* is any positive real number other than one, the base ***a*** exponential function $f(x)=a^x$ is one-to-one. Therefore, it has an inverse.
- This inverse is what we call the ***base a logarithm function.***

Base *a* Logarithm Function

- Definition: The ***base a logarithm function*** $y = log_a x$ is the inverse of the base *a* exponential function $y=a^x$ $(a > 0, a\neq 1)$
- Domain: The domain of $y = log_a x$ is $(0, \infty)$, the range of a^x.
- Range: The range of $y = log_a x$ is $(-\infty,\infty)$, the domain of a^x.

log(x) and ln(x)

- Logarithms with base 10 and base *e* are used most commonly, so they have their own special notation and names.
- $\log_e x = \ln x$ → *natural log*
- $\log_{10} x = \log x$ → common log

Properties of logarithms

- Inverse Properties:
- Base a:
  - $a^{\log_a x} = x$, a > 1, x > 0
  - $\log_a a^x = x$, x>0
- Base e:
  - $e^{\ln x} = x$, x > 0
  - $\ln e^x = x$

$$\begin{tikzcd} f(x) = \boxed{a^x} & f^{-1}(x) = \boxed{\log_a x} \arrow[l, bend right] \arrow[l, bend left, leftarrow] \end{tikzcd}$$

$$f(f^{-1}(x)) = x$$

$$f^{-1}(f(x)) = x$$

Example 4 (Similar)

- Solve for *x*.

a) $\boxed{\ln x} = (8y^2 - 3)$

$e \qquad e$

$x = e^{8y^2-3}$

$\log_e e^1 = \boxed{1}$

b) $e^{2x} = 8$

$\ln(e^{2x}) = \ln 8$

$2x \boxed{\ln e} = \ln 8$

$\frac{2x}{2} = \frac{\ln 8}{2}$

$x = \frac{\ln 8}{2} \rightarrow \frac{3 \ln 2}{2}$

$\ln 2^3$

Extra Example

- Solve for x: $4^x + 4^{-x} = 10$

$\ln ab = \ln a + \ln b$

$4^x(4^x + 4^{-x}) = (10)4^x$

$2^3 \cdot 2^3 =$

$2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 = 2^6$

$(4^x)^2$

$4^{2x} + \overset{1}{\cancel{4^0}} = 10(4^x)$

$\rightarrow 4^{2x} - 10(4^x) + 1 = 0$

$u^2 - 10u + 1 = 0$

$u = \frac{10 \pm \sqrt{[illegible]96}}{2} = \frac{\overset{5}{\cancel{10}} \pm \overset{2}{\cancel{4}}\sqrt{6}}{\cancel{2}}$

$u = 4^x$

$u = 5 \pm 2\sqrt{6}$

$\ln 4^x = \ln(5 \pm 2\sqrt{6})$

$\frac{x \cancel{\ln 4}}{\cancel{\ln 4}} \boxed{= \frac{\ln(5 \pm 2\sqrt{6})}{\ln 4}}$

More Properties of Logarithms

- For any real numbers $x>0$ and $y>0$,
- Product Rule:
  - $\log_a xy = \log_a x + \log_a y$
- Quotient Rule:
  - $\log_a (x/y) = \log_a x - \log_a y$
- Power Rule:
  - $\log_a x^y = y \log_a x$

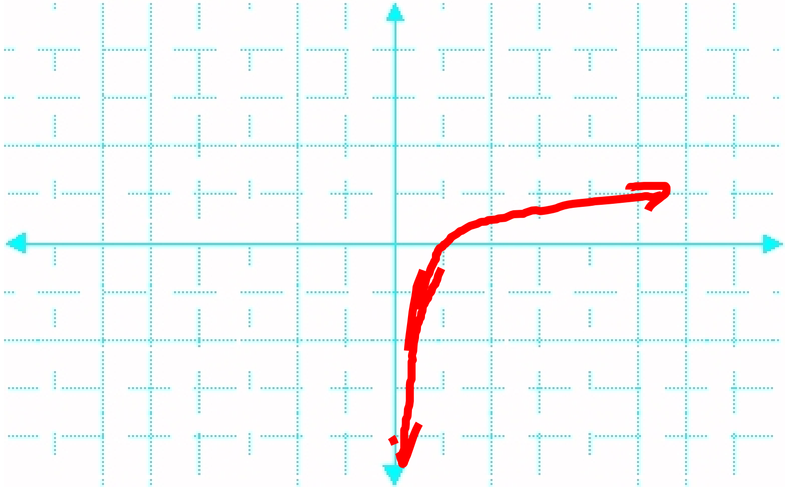
Changing Bases

- If we need to evaluate $\log_a x$ for any base $a>0$, $a\neq 1$, we can use the following formula:
- Change of Base Formula:

$$\log_a x = \frac{\ln x}{\ln a}$$

Example 5 (Similar)

- Graph *f(x)* = $\log_7 x$
- Hint: Use the formula $\log_a x = \dfrac{\ln x}{\ln a}$ and your graphing calculator.



Applications

- You invest $2000 in an account that earns 4.75% interest compounded annually. How long will it take until your money is doubled?

$$\frac{4\cancel{000}\ \cancel{y}}{2\cancel{00}} = \frac{\boxed{\cancel{2000}}\boxed{(1+.0475)^{t}}}{2\cancel{000}} \leftarrow$$

$$\ln 2 = \ln(1+.0475)^{t}$$

$$\frac{\ln 2}{\ln(1.0475)} = \frac{t(\ln(1.0475))}{\ln(1.0475)}$$

$$t = 14.94 \text{ years}$$

Applications

- The population of Seneca is 2100 and increasing by 1.65% each year. Predict when the population will reach 10,000.

Example 7

- The table shows the number of metric tons of oil produced by Indonesia for 3 different years.
- Find the natural logarithm regression equation for the data in the table and use it to estimate the number of metric tons of oil produced by Indonesia in 1982 and 2000.
- Enter data in L1 and L2
- STAT,>Calc, 9 (LnReg)
- Y1=VARS, 5, >>EQ, 1 RegEQ
- Set up window & graph
- 2nd CALC, 1 to check for 1982 and 2000

Indonesia's Oil Production

	Year	Metric Tons-in millions
0	1960→	20.56
10	1970	42.10
22	1982	59.5
30	1990	70.10
40	2000	83.5

Assignment

- Sec 1.5: 37-49
    - Follow directions
    - Solve algebraically unless directed otherwise