1.5 Functions and Logarithms

Goals

- To identify a one-to-one function
- To determine the algebraic representation and the graphical representation of a function and its inverse
- To apply properties of logarithms
- To use logarithmic regression to solve problems

One-to-One Functions

- A function *f(x)* is **one-to-one** on a domain *D* if *f(a) ≠ f(b)* whenever *a ≠ b.*
- *How do we know if a function is one-to-one?*
- *Easiest way…graph it!*

Horizontal Line Test

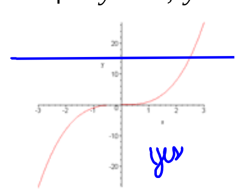
- The graph of a one-to-one function *y = f(x)* can intersect any horizontal line no more than one time.
- If it intersects the graph more than once, then two different x values have the same y value and therefore is not one-to-one.
- (One-to-one: one x, one y).

Example 1

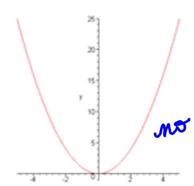
- Graph $y = x^3,\ y = x^2,\ y = \sqrt{x},\ y = |x|$
- Which of these functions are one-to-one?

Example 1

- Graph $y = x^3,\ y = x^2$



yes

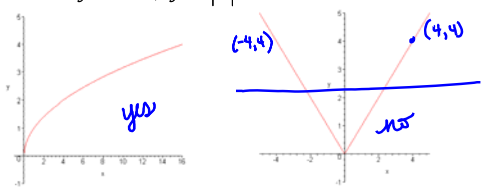


no

- Which of these functions are one-to-one?

Example 1

- Graph $y = \sqrt{x},\ y = |x|$



- Which of these functions are one-to-one?

$y = \sqrt{x}$

Inverses

- Since every output of a one-to-one function comes from only one input, we can reverse the function to send the outputs back to the inputs from which they came.
- The ***inverse of f*** is defined by reversing a one-to-one function *f*.
- The domain and range of inverse functions are reversed (or switched or transposed)

Notation

- The symbol for the inverse of *f* is f^{-1}, which is read, "f inverse."
- Important note: the -1 in f^{-1} is **not** an exponent
  - [$f^{-1}(x)$ does not mean $(1/f(x))$]

Compositions

$f(3) = 27 \quad g(27) = 3$

$f(-2) = -8 \quad g(-8) = -2$

- $f(x) = x^3$; $g(x) = x^{1/3}$
- Find $f(g(x))$ and $g(f(x))$

$f(x) = x^3$; $f(x^{1/3}) = (x^{1/3})^3 = x^1$

$g(x) = x^{1/3}$ $g(x^3) = (x^3)^{1/3} = x$

$f(3) = 3 \quad g(3) = \frac{1}{3}$

- $f(x) = x$; $g(x) = 1/x$
- Find $f(g(x))$ and $g(f(x))$

$f(x) = x \quad f(\frac{1}{x}) = \frac{1}{x}$

$g(x) = \frac{1}{x} \quad g(x) = \frac{1}{x}$

Inverse Functions

- The result of composing a function and its inverse in either order is the **identity function**, which is the function that assigns each number to itself. $f(x) = x$

Finding Inverses

- To find the inverse of a function *f*
  - Solve the equation $y = f(x)$ for *x* in terms of *y*.
  - Interchange *x* and *y*. The resulting formula will be $y = f^{-1}(x)$.

$f(x) = x^3$

$\sqrt[3]{y} = \sqrt[3]{x^3}$

$\sqrt[3]{y} = x$

$\sqrt[3]{x} = y$ ($f^{-1}(x)$)

Domain and Range of Inverses

- The domain of f^{-1} is the range of f.
- The range of f^{-1} is the domain of f.

Example 2

- Show first that y = f(x) = -2x + 4 is one-to-one. Then find its inverse function.

It is one-to-one (passes the horiz. line test)

$y = -2x + 4$
$-4 \qquad -4$

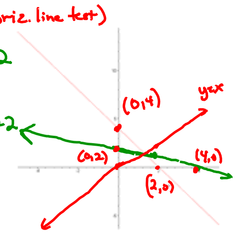
$\frac{y-4}{-2} = \frac{-2x}{-2}$

$\frac{y-4}{-2} = x$

$-\frac{1}{2}y + 2 = x$

$y = -\frac{1}{2}x + 2$

$f^{-1}(x) = -\frac{1}{2}x + 2$

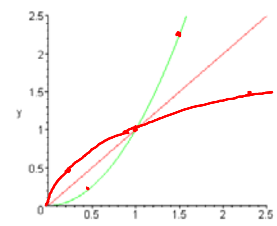


Graphing the Inverse

- If a one-to-one function is reflected over the line *y=x*, the resulting function is the original function's inverse.
- Reminder:
  - If (a,b) is a point on your original graph, (b,a) will be a point on its inverse's graph.

Example 3

- Graph the inverse of $f(x) = x^2$ where $x \geq 0$.



$f(1) = 1 \qquad f^{-1}(1) = 1$

$f(.5) = .25 \qquad f^{-1}(.25) = .5$

$f(1.5) = 2.25 \qquad f^{-1}(2.25) = 1.5$

$\sqrt{y} = \sqrt{x^2}$

$\sqrt{y} = x$

$\sqrt{x} = y \qquad f^{-1}(x) = \sqrt{x}$

Assignment

- Go back and read Sec1.5
  - p. 39: 1-6 all, 8-24 even
- Show problem, work if applicable, and answer(s)
- Quiz Sec 1.2 & 1.3 Next time