Sec 1.7 Solve Absolute Value Equations and Inequalities

Before You solved linear equations and inequalities.

Now You will solve absolute value equations and inequalities.

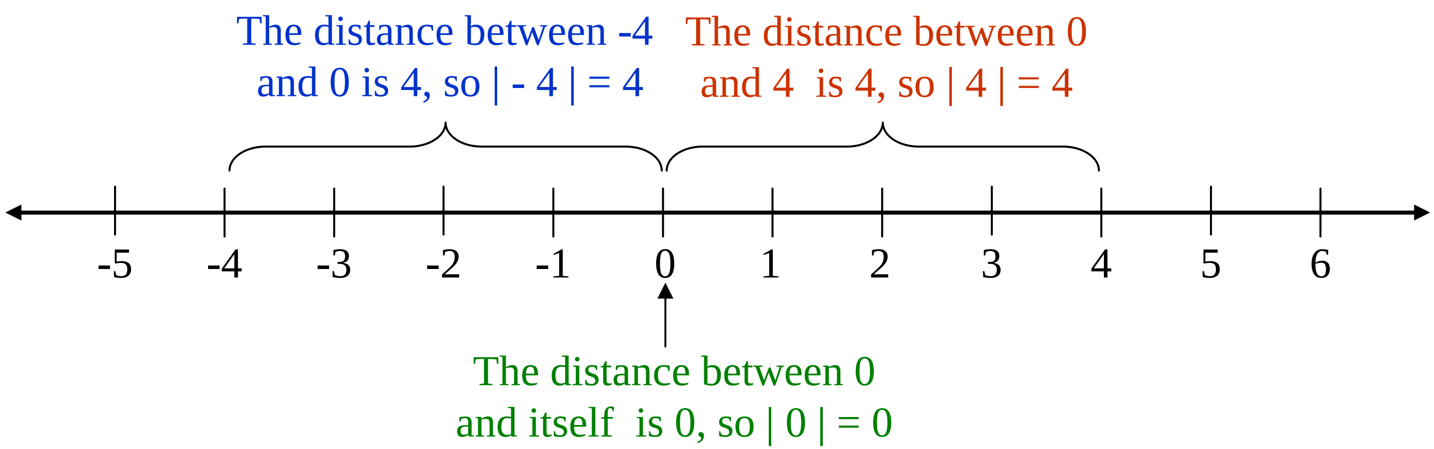
Why? So you can describe hearing ranges of animals, as in Ex. 81.

Goals

- **Goal 1: To solve absolute value equations and inequalities**
- **Goal 2: To use absolute value equations and inequalities to solve real life problems**

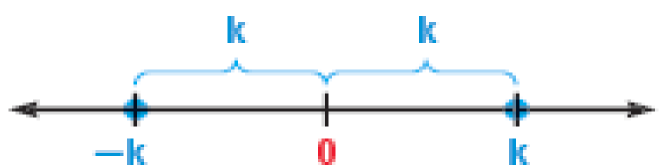
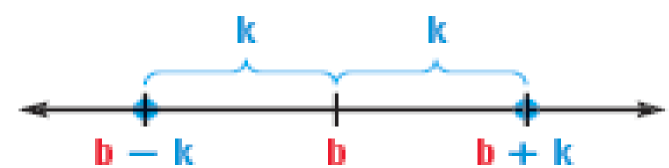
Geometric Definition

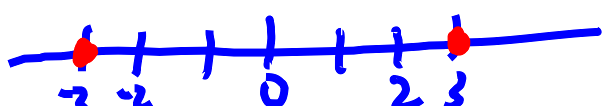
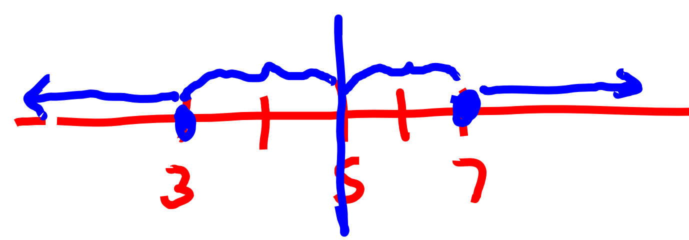
- The **<u>absolute value</u>** of a number *x*, written |*x*|, is the distance the number is from zero on a number line.
- Notice that the absolute value of a number is always nonnegative.



KEY CONCEPT — *For Your Notebook*

Interpreting Absolute Value Equations

Equation	$\lvert x\rvert = \lvert x - 0\rvert = k$	$\lvert x - b\rvert = k$
Meaning	The distance between x and 0 is k.	The distance between x and b is k.
Graph		
Solutions	$x - 0 = -k$ or $x - 0 = k$ $x = -k$ or $x = k$	$x - b = -k$ or $x - b = k$ $x = b - k$ or $x = b + k$

Equation	$\lvert x\rvert = 3$ $\lvert x - 0\rvert = 3$	$\lvert x - 5\rvert = 2$
Meaning	Distance between x and 0 is 3	Distance betw. x & 5 is 2
Graph		
Solutions	$x = 3, -3$	

Solving an Absolute Value

- To solve the absolute value $\left|ax+b\right|=c$, use two equations. One if the inside is positive or zero, $ax+b=c$. The other if the inside is negative $-(ax+b)=c$.

Algebraic Definition

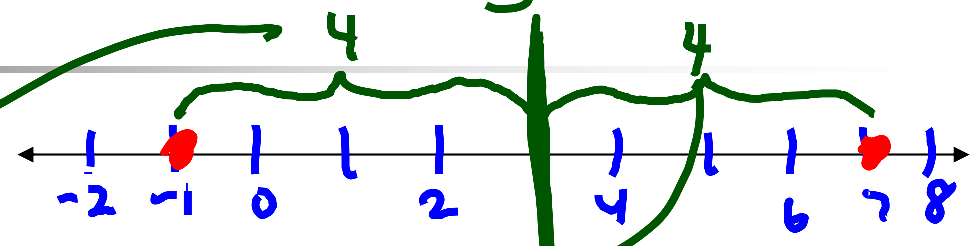
$$|x| = \begin{cases} x, & \text{if } x \text{ is positive} \\ 0, & \text{if } x = 0 \\ -x, & \text{if } x \text{ is negative} \end{cases}$$

$$|x| = \begin{cases} x, & x \geq 0 \\ -x, & x < 0 \end{cases}$$

Disjunct

Example 1

$$\left| x - 3 \right| = 4$$



- When inside is positive, absolute value doesn't affect it.

$$\boxed{x - 3 = 4}$$
$$+3 \quad +3$$
$$x = 7$$

OR

- When inside is negative, absolute value takes the opposite of the inside.

$$\frac{-1(x-3)}{-1} = \frac{4}{-1}$$
$$x - 3 = -4$$
$$+3 \quad +3$$
$$x = -1$$

$|3(x-2)| = 12$

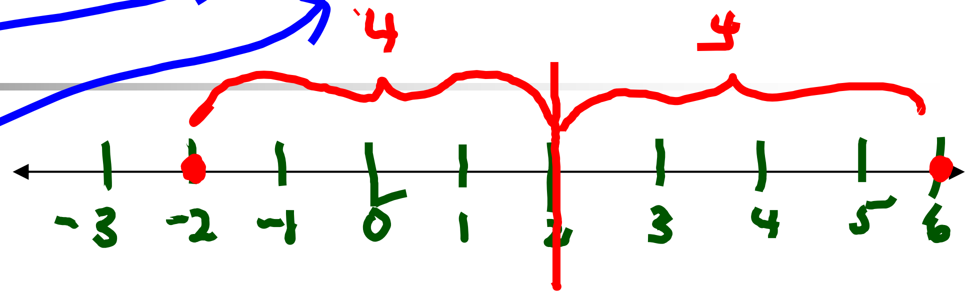
$|3|\,|x-2| = 12$

$\frac{3|x-2|}{3} = \frac{12}{3}$

$|x-2| = 4$

Example 2

$$|3x-6| = 12$$



- When inside is positive, absolute value doesn't affect it.
- When inside is negative, absolute value takes the opposite of the inside.

$3x - 6 + 6 = 12 + 6$

$\frac{3x}{3} = \frac{18}{3}$

$x = 6$

$-(3x-6) = 12$

$-3x + 6 - 6 = 12 - 6$

$\frac{-3x}{-3} = \frac{6}{-3}$

$x = -2$

Extraneous Solutions

EXTRANEOUS SOLUTIONS When you solve an absolute value equation, it is possible for a solution to be *extraneous*. An **extraneous solution** is an apparent solution that must be rejected because it does not satisfy the original equation.

Example 3

$|24| = 24$

$\cancel{|8| = -8}$

$$|2x + 12| = 4x$$

- When inside is positive, absolute value doesn't affect it.

$$\boxed{2x + 12 = 4x}$$
$$-2x \qquad -2x$$
$$12 = 2x$$
$$6 = x$$

- When inside is negative, absolute value takes the opposite of the inside.

$$\boxed{-(2x + 12) = 4x}$$
$$+1 \qquad -1$$
$$2x + 12 = -4x$$
$$-2x \qquad -2x$$
$$\frac{12}{-6} = \frac{-6x}{-6}$$
$$\cancel{-2 = x}$$

$x = \cancel{-2}$ or 6

GUIDED PRACTICE

$$|x + 2| = 7$$

$x + 2 = 7$

$-2 \quad -2$

$x = 5$

$-(x+2) = 7$

$-x - 2 = 7$

$+2 \quad +2$

$-x = 9$

$x = -9$

$-(-9+2)$

$$|4x - 1| = 2x + 9$$

$4x - 1 = 2x + 9$

$-2x \quad +1 \quad -2x \quad +1$

$2x = 10$

$x = 5$

$-(4x-1) = 2x+9$

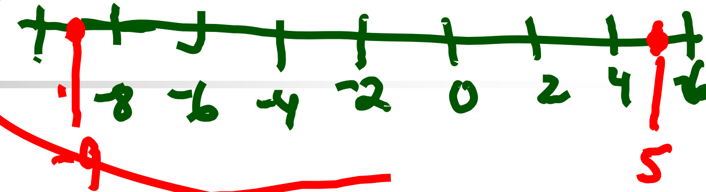
$-4x + 1 = 2x + 9$

$+4x \quad -9 \quad +4x \quad -9$

$-8 = 6x$

$-4/3 = x$

$$\boxed{|2x + 5| = 3x}$$



$2x + 5 = 3x$

$-2x \quad -2x$

$5 = x$

$-(2x+5) = 3x$

$-2x - 5 = 3x$

$-5 = 5x$

$\frac{-5}{5} = \frac{5x}{5}$

$\cancel{-1 = x}$

$|19| = 19$

$|4(-\tfrac{4}{3}) - 1| = 2(-\tfrac{4}{3}) + 9$

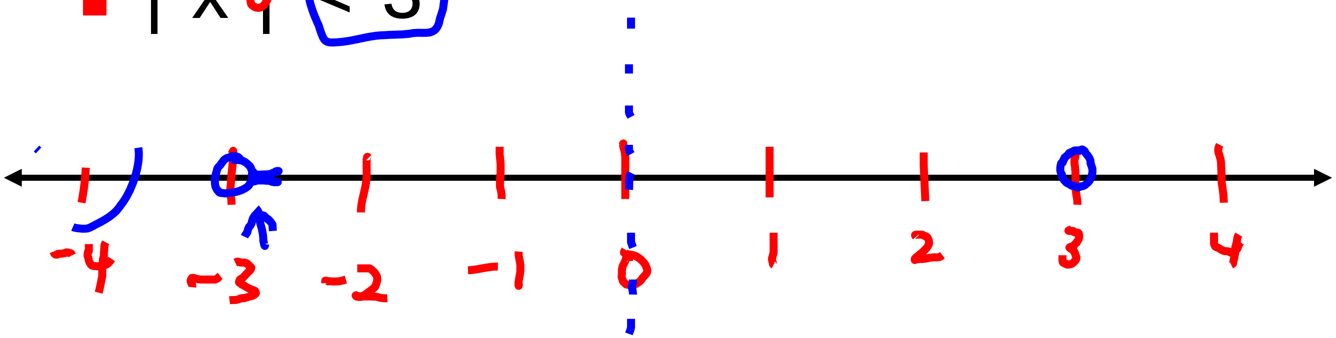
$|-\tfrac{16}{3} - \tfrac{3}{3}| = -\tfrac{8}{3} + \tfrac{27}{3}$

$\tfrac{19}{3}$

$\boxed{|-\tfrac{19}{3}| = \tfrac{19}{3}}$

Absolute Value Inequalities

- Graph the solution to the following:
  - $| x - 0 | < 3$



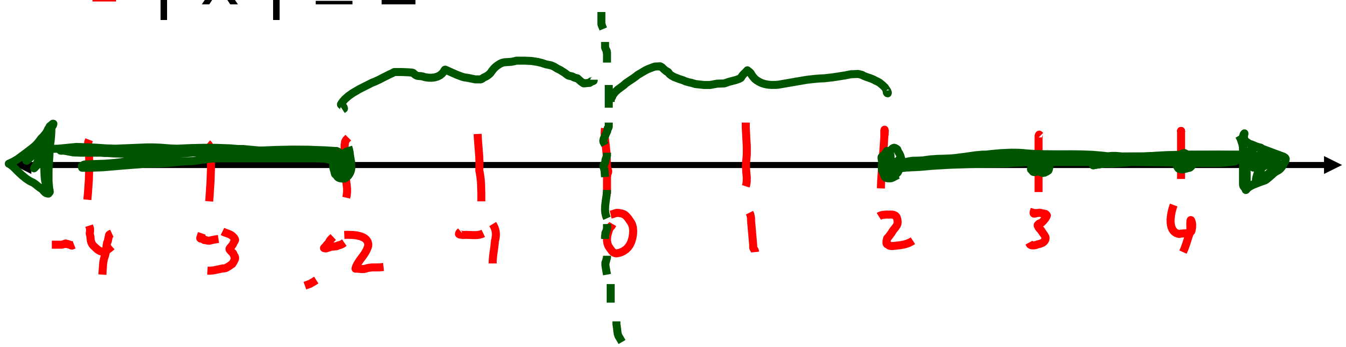
- Does this give a conjunction or a disjunction?
  - Conjunction

Conjunction

Less th AND

Absolute Value Inequalities

- Graph the solution to the following:
  - $| x-0| \geq 2$



- Does this give a conjunction or a disjunction?
  - Disjunction

GreatOR

Absolute Value Inequalities

$$|ax+b| < c$$

- This inequality is a conjunction.
- When the inside (ax+b) is negative, the absolute value will take the opposite of the inside

$$-(ax+b) < c$$

- When the inside (ax+b) is positive, the absolute value will not change the inside

$$(ax+b) < c$$

Absolute Value Inequalities

$$|ax+b| < c$$

- These can be written as a compound inequality.

$$-(ax+b) < c \text{ and } (ax+b) < c$$

$+1 \qquad -1$

  - Get the ax+b in the first inequality by itself by multiplying both sides by -1.

$$(ax+b) > -c \text{ and } (ax+b) < c$$

  - Since both inequalities are now in terms of ax+b, rewrite it without “and”

$$-c < ax+b < c$$

Example 4: Solve

$$|2x+7| < 11$$

$2x + 7 < 11$ and $-(2x+7) < 11$

$-7 \quad -7$

$2x < 4$

$x < 2$

$-2x - 7 < 11$

$+7 \quad +7$

$\frac{-2x}{-2} < \frac{18}{-2}$

$\boxed{x > -9}$

$x < 2$ and $x > -9$

$\boxed{-9 < x} < 2$

$-11 < 2x + 7 < 11$

$-7 \quad -7 \quad -7$

$\frac{-18}{2} < \frac{2x}{2} < \frac{4}{2}$

$\boxed{-9 < x < 2}$

Absolute Value Inequalities

$$|ax+b|>c$$

- This inequality is a disjunction.
- When the inside (ax+b) is negative, the absolute value will take the opposite of the inside

$$-(ax+b)>c$$

- When the inside (ax+b) is positive, the absolute value will not change the inside

$$(ax+b)>c$$

Absolute Value Inequalities

$$|ax+b| > c$$

- These can be written as a compound inequality.

$$\underbrace{-(ax+b)}_{-1} > \underbrace{c}_{-1} \text{ or } (ax+b) > c$$

- Get the ax+b in the first inequality by itself by multiplying both sides by -1.

$$\boxed{(ax+b) < -c} \text{ or } \boxed{(ax+b) > c}$$

Example 5: Solve

$$|3x-2| \geq 8 \quad \text{or}$$

$3x - 2 \geq 8$ or $-(3x-2) \geq 8$

$\quad +2 \quad +2$

$3x \geq 10$

$x \geq \frac{10}{3}$

$-3x + 2 \geq 8$

$\quad -2 \quad -2$

$\frac{-3x}{-3} \geq \frac{6}{-3}$

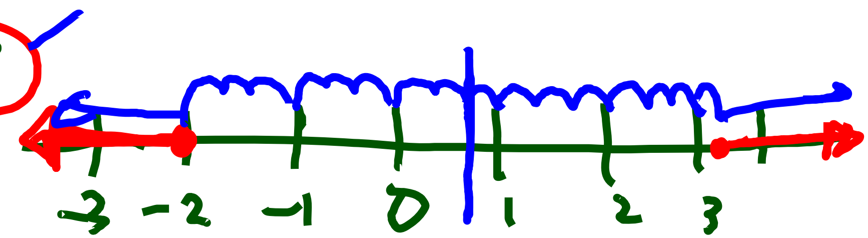
$x \leq -2$

$|3(x - \frac{2}{3})| \geq 8$

$\cancel{3}|x - \frac{2}{3}| \geq 8/3$

$|x - \frac{2}{3}| \geq \frac{8}{3}$

$x \leq -2$ or $x \geq \frac{10}{3}$



Less Than AND   Great OR

GUIDED PRACTICE

$|x+4| \ge 6$

$x+4 \ge 6$ OR $-(x+4) \ge 6$

$-4 \quad -4$

$x \ge 2$

$-x-4 \ge 6$

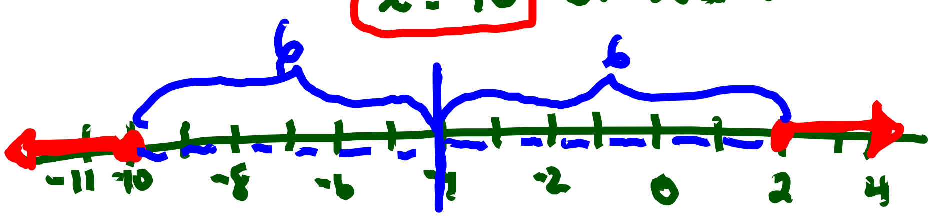
$+4 \quad +4$

$-x \ge 10$

$-1 \quad -1$

$x \le -10$

$\boxed{x \le -10}$ or $x \ge 2$



$|2x+1| \le 9$

$2x+1 \le 9$ and $-(2x+1) \le 9$

$-1 \quad -1$

$2x \le 8$

$x \le 4$

$-2x-1 \le 9$

$+1 \quad +1$

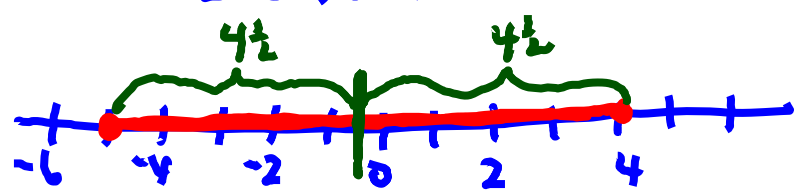
$-2x \le 10$

$-2 \quad -2$

$x \ge -5$

$x \ge -5$ and $x \le 4$

$-5 \le x \le 4$



Example 6 Tolerance

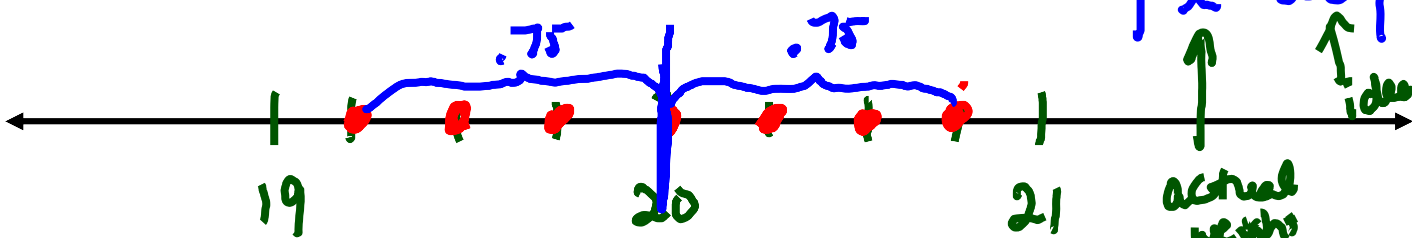
- A cereal manufacturer has a tolerance of 0.75 ounce for a box of cereal that is supposed to weigh 20 ounces. Write and solve an absolute value inequality that describes the acceptable weights for "20 ounce" boxes of cereal.
- We can show acceptable weights using a graph.

$$|x - 20| \le .75$$

actual weights

ideal

tolerance



Example 6 Tolerance

- A cereal manufacturer has a tolerance of 0.75 ounce for a box of cereal that is supposed to weigh 20 ounces. Write and solve an absolute value inequality that describes the acceptable weights for "20 ounce" boxes of cereal.
- The amount that the weight is off can be found by subtracting the ideal weight (20 oz) from the actual weight.
- This difference should be less than the tolerance.

$$\left| \boxed{\text{Actual Weight}} - \boxed{\text{Ideal Weight}} \right| \leq \boxed{\text{Tolerance}}$$

Example 6 Tolerance

- A cereal manufacturer has a tolerance of 0.75 ounce for a box of cereal that is supposed to weigh 20 ounces. Write and solve an absolute value inequality that describes the acceptable weights for "20 ounce" boxes of cereal.
- Why do we use the absolute value bars here?

$\cancel{21} - 20 \leq .75$

$1 \leq .75$

$19 - 20 \leq .75$

$|-1| \leq .75$

$1 \leq .75$

$$\left|\ \boxed{\text{Actual Weight}} - \boxed{\text{Ideal Weight}}\ \right| \leq \boxed{\text{Tolerance}}$$

Example 6 Tolerance

- A cereal manufacturer has a tolerance of 0.75 ounce for a box of cereal that is supposed to weigh 20 ounces. Write and solve an absolute value inequality that describes the acceptable weights for "20 ounce" boxes of cereal.
- Give labels to our values
  - Actual weight = x (ounces)
  - Ideal weight = 20 (ounces)
  - Tolerance = 0.75 (ounces)

Example 6 Tolerance

- A cereal manufacturer has a tolerance of 0.75 ounce for a box of cereal that is supposed to weigh 20 ounces. Write and solve an absolute value inequality that describes the acceptable weights for "20 ounce" boxes of cereal.
- Substitute labels in to verbal model to create algebraic model
  - $| x - 20 | \leq 0.75$

$x - 20 \leq .75$ and $-(x-20) \leq .75$

$+20 \quad +20$

$x \leq 20.75$

$-x + 20 \leq .75$

$-20 \quad -20$

$\frac{-x}{-1} \leq \frac{-19.25}{-1}$

$x \geq 19.25$

Example 6 Tolerance

- Solve
  - $| x - 20 | \leq 0.75$

$x - 20 \leq .75$ and $-(x-20) \leq .75$

$+20 \quad +20$

$x \leq 20.75$

$-x + 20 \leq .75$

$-20 \quad -20$

$-x \leq -19.25$

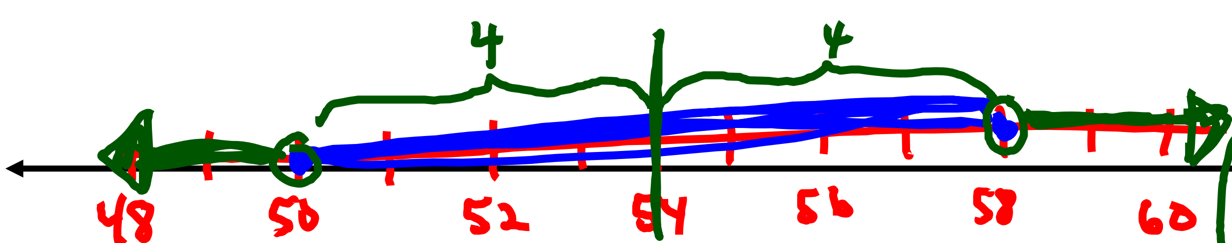
$\overline{-1} \quad \overline{-1}$

$x \geq 19.25$

$19.25 \leq x \leq 20.75$

Example 7 Quality Control

- QUALITY CONTROL. You are a quality control inspector at a bowling pin company. A regulation pin must weight between 50 ounces and 58 ounces, inclusive. Write an absolute value inequality describing the weights you should reject.
- We can show acceptable weights using a graph.



$|x-54| \le 4$

$\boxed{|x-54| > 4}$

actual ideal

Example 7 Quality Control

- QUALITY CONTROL. You are a quality control inspector at a bowling pin company. A regulation pin must weight between 50 ounces and 58 ounces, inclusive. Write an absolute value inequality describing the weights you should reject.
- But we are looking for weights we should reject. What would that graph look like?
  - (See green graph from previous slide)

Example 7 Tolerance

- QUALITY CONTROL. You are a quality control inspector at a bowling pin company. A regulation pin must weight between 50 ounces and 58 ounces, inclusive. Write an absolute value inequality describing the weights you should reject.
- What do you think the "ideal" weight would be in this problem and how do you "calculate" it? $\frac{50+58}{2} = 54$
- What is the tolerance then? $58 - 54 = 4$
- Write a verbal model for this problem.

$$\left|\ \text{Weight of Pin} - \text{Ideal Weight}\ \right| > \text{Tolerance}$$

Example 7 Tolerance

- QUALITY CONTROL. You are a quality control inspector at a bowling pin company. A regulation pin must weight between 50 ounces and 58 ounces, inclusive. Write an absolute value inequality describing the weights you should reject.
- Give labels to the parts of the verbal model.
  - Weight of pin = w (ounces)
  - Average of extreme weights (ideal weight) = (50+58)/2 = 54 (ounces)
  - Tolerance = 58-54 = 4 (ounces)

Example 7 Tolerance

- QUALITY CONTROL. You are a quality control inspector at a bowling pin company. A regulation pin must weight between 50 ounces and 58 ounces, inclusive. Write an absolute value inequality describing the weights you should reject.
- Create the algebraic model:
  - $| w - 54 | > 4$

Example 7 Tolerance

- Solve
  - $|\boxed{w - 54}| > 4$

$w - 54 > 4$ OR $-(w-54) > 4$

$+54 \quad +54$

$w > 58$ or

$-w + 54 > 4$

$-54 \quad -54$

$\frac{-w}{-1} > \frac{-50}{-1}$

$w < 50$

$|\text{accept.} - \text{ideal}| \boxed{\begin{smallmatrix} \le \\ \ge \end{smallmatrix}}$ tolerance

Closure

- $|ax + b| < c$ results in a conjunction
  - What conjunction is it?

$-c < ax + b < c$

- $|ax + b| > c$ results in a disjunction
  - What disjunction is it?

$ax + b > c \quad ax + b < -c$

Assignment

- Sec 1.7
  - 1-5, 9-37 LC, 43-59 LC, 74-77