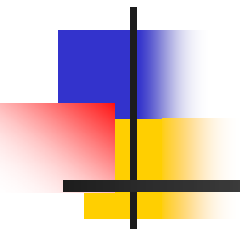
Sec 1.6 Solve Linear Inequalities



Before You solved linear equations.

Now You will solve linear inequalities.

Why? So you can describe temperature ranges, as in Ex. 54.

Goals

- **Goal 1: To solve simple inequalities**
- **Goal 2: To solve compound inequalities**

Equations and Inequalities

- Equations and inequalities are both statements comparing two expressions.
- In this section, we will learn the similarities and differences, and we will learn to solve inequalities, just as you already know how to solve equations.

Definition

- Inequalities such as $x \leq 1$ and $2n + 3 > 9$ are examples of **linear inequalities** in one variable.

Inequalities

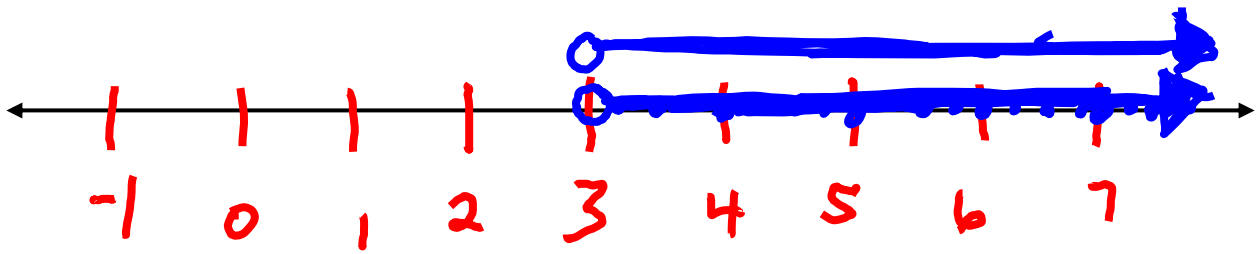
- A **linear inequality** in one variable can be written in one of the following forms, where *a* and *b* are real numbers and *a* is not equal to zero:
- $ax+b<0$
- $ax+b>0$
- $ax+b\leq 0$
- $ax+b\geq 0$

Inequalities

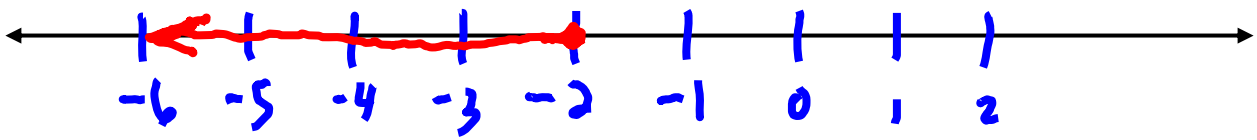
- A **solution** of an inequality in one variable is a value that, when substituted for the variable, results in a true statement.
- Give some solutions to the inequality $x<5$.
- The graph of an inequality in one variable consists of all points on the number line that represents solutions.

Graphing Inequalities

- The graph of $x>3$ is



- The graph of $x\leq -2$ is



Compound Inequalities

- Sometimes we want more than one inequality to be true at once.
- For example the room temperature is comfortable if it is more than 68° and it is less than 78°
- A **compound inequality** is two simple inequalities joined by “and” or “or”.

Conjunctions/Example 2a

- A compound inequality joined by “and” is called a conjunction. There is a special way that only a conjunction can be written.

$$-2 \leq x < 1$$

- Note here that x is BETWEEN -2 and 1.
- Always put the smaller # on the left and the larger on the right and use < or ≤ symbols.
- The graph would look like:

Conjunctions

$$-2 \leq x < 1$$

- Written as a conjunction of two inequalities using the word “and” this compound inequality would be:

$$-2 \leq x \text{ and } x < 1$$

$$\text{(or) } x \geq -2 \text{ and } x < 1$$

Disjunctions/Example 2b

- A compound inequality joined by “or” is called a disjunction. A disjunction must have two separate inequalities and the word “or”.

$$x < -1 \text{ or } x \geq 2$$

- The graph would look like:

GUIDED PRACTICE for Examples 1 and 2

Graph the inequality.

1. $x > -5$

2. $x \leq 3$

3. $-3 \leq x < 1$

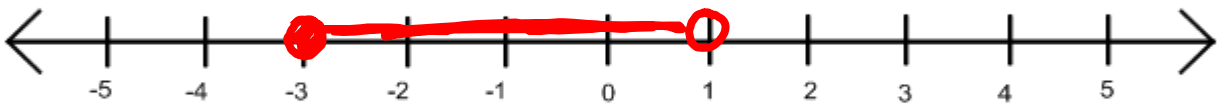
4. $x < 1$ or $x \geq 2$

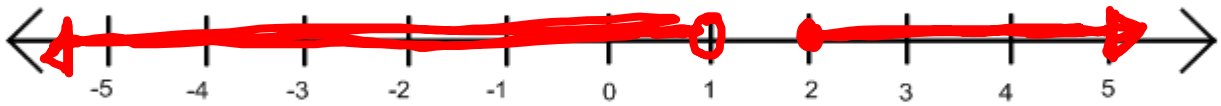
open

closed , open

x is between -3 and 1

o≠3





2 > x

Inequalities

- Write two true inequalities involving integers, one using < and one using >.
- Add, subtract, multiply, & divide each side of the inequality by 2 and –2. Highlight the ones that are true.

	2 < 6		8 > −4	
+	4 < 8	0 < 4	10 > −2	6 > −6
−	0 < 4	4 < 8	6 > −6	10 > −2
×	4 < 12	−4 < −12	16 > −8	−16 > −8
÷	1 < 3	−1 < −3	4 > −2	−4 > 2
	2	−2	2	−2

Inequalities

- What general conclusion can you make about the operations you can perform on a true inequality to produce another true inequality.

Add or subtract anything

Mult. or ÷ by positive

Transformations

- Just as equations had transformations that we used to help solve them (Addition Property of Equality and Multiplication Property of Equality are two of them), so do inequalities.
- Here are the transformations that produce equivalent inequalities.

Transformations

- *Add* the same number to both sides.
- *Subtract* the same number from both sides.
- *Multiply* both sides by the same *positive* number.
- *Divide* both sides by the same *positive* number.
- *Multiply* both sides by the same *negative* number and *reverse* the inequality.
- *Divide* both sides by the same *negative* number and *reverse* the inequality.

Example 3

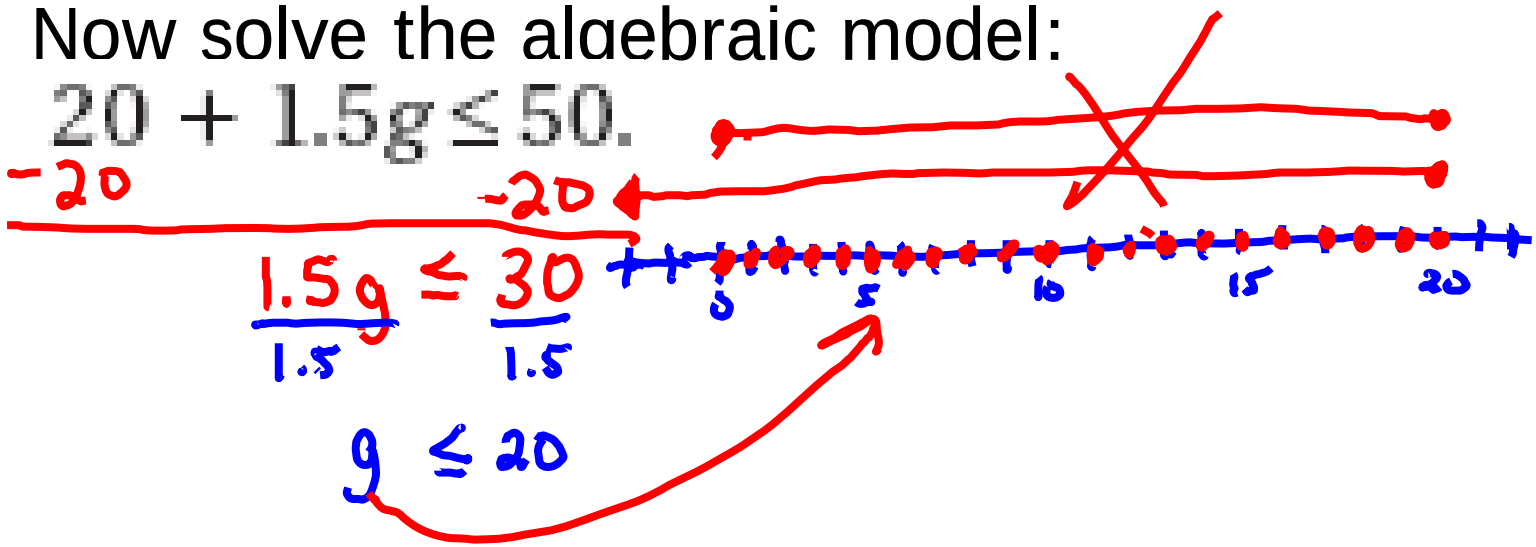
- **FAIR** You have $50 to spend in the county fair. You spend $20 for admission. You want to play a game that costs $1.50. Describe the possible numbers of times you can play the game.
- First create a verbal model then add labels:

Admission fee (dollars)	+	Cost per game (dollars/game)	•	Number of games (games)	≤	Amount you can spend (dollars)
20	+	1.50	•	g	≤	50

Example 3

- **FAIR** You have $50 to spend in the county fair. You spend $20 for admission. You want to play a game that costs $1.50. Describe the possible numbers of times you can play the game.
- Now solve the algebraic model:

$$20 + 1.5g \leq 50.$$



Example 4

X or ÷ by − flip ≥ to ≤

- Solve and graph.

$$3x + 2 \geq 5x - 1$$

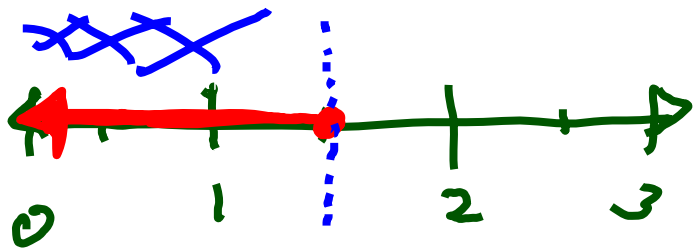
$-5x \quad -2 \qquad -5x \quad -2$

$\frac{-2x}{-2} \geq \frac{+3}{+2}$

$x \leq \frac{3}{2}$ 1.5

$\frac{3}{2} \geq \frac{2x}{2}$

$\frac{3}{2} \geq x \rightarrow x \leq \frac{3}{2}$



GUIDED PRACTICE for Examples 3 and 4

Solve the inequality. Then graph the solution.

5. $4x + 9 < 25$ **6.** $1 - 3x \geq -14$ **7.** $5x - 7 \leq 6x$ **8.** $3 - x > x - 9$

Example 5

- Solve and graph.

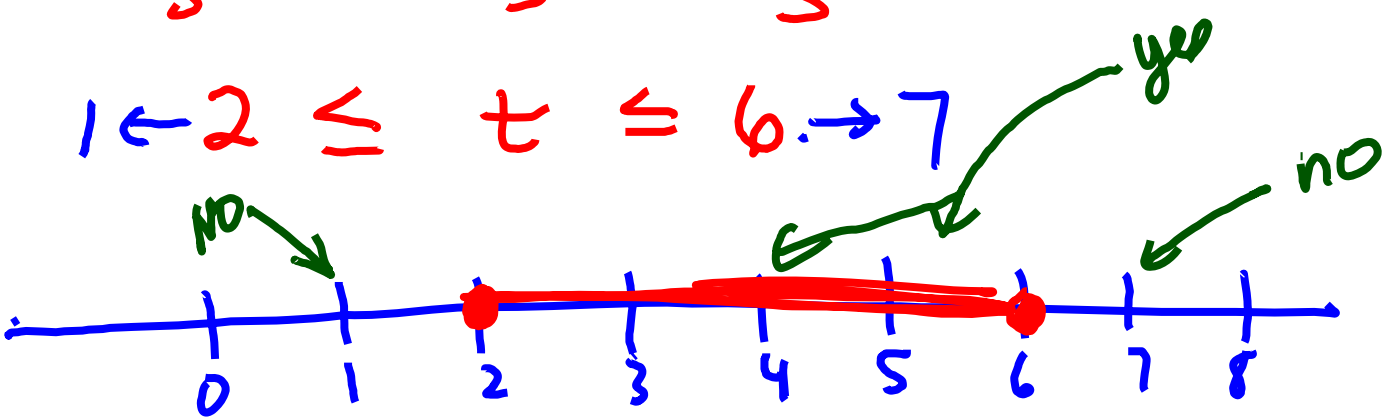
$$-2 \le 3t - 8 \le 10$$

+8 +8 +8

$$6 \le 3t \le 18$$

$$\frac{6}{3} \le \frac{3t}{3} \le \frac{18}{3}$$

$$1 \leftarrow -2 \le t \le 6 \rightarrow 7$$



$-2 \le -5 \le 10$ X

$-2 \le 4 \le 10$ ✓

$-2 \le 13 \le 10$ X

Example 6

- Solve and graph.

$2x + 3 < 5$ or $4x - 7 > 9$

$-3 \quad -3$

$\frac{2x}{2} < \frac{2}{2}$

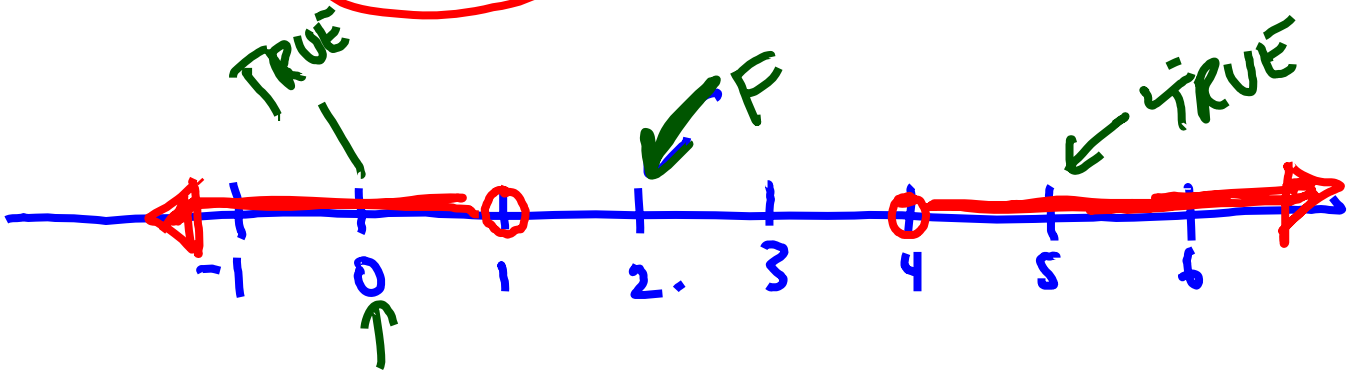
$x < 1$

$+7 \quad +7$

$\frac{4x}{4} > \frac{16}{4}$

$x > 4$

OR



GUIDED PRACTICE for Examples 5, 6

Solve the inequality. Then graph the solution.

9. $-1 < 2x + 7 < 19$

10. $-8 \leq -x - 5 \leq 6$

11. $x + 4 \leq 9$ or $x - 3 \geq 7$

12. $3x - 1 < -1$ or $2x + 5 \geq 11$

10) $-8 \leq -x - 5 \leq 6$

① $+5 \quad +5 \quad +5$

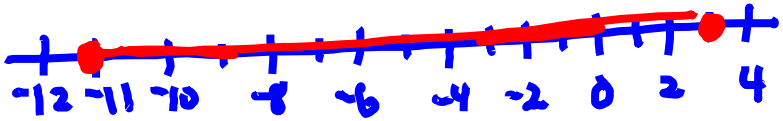
$-3 \leq -x \leq 11$

② $\div -1 \quad \div -1 \quad \div -1$ FLIP!

$3 \geqslant x \geqslant -11$

Rewrite small to large:

$-11 \leqslant x \leqslant 3$



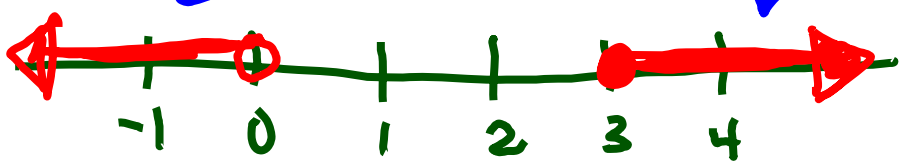
12) $3x - 1 < -1$ or $2x + 5 \geqslant 11$

$+1 \quad +1$ $\quad$ $-5 \quad -5$

$3x < 0$ $\quad$ $2x \geqslant 6$

$\div 3 \quad \div 3$ $\quad$ $\div 2 \quad \div 2$

$x < 0$ or $x \geqslant 3$



Example 7

- The weight w (in pounds) of an Icelandic saithe is given by $w = 10.4t - 2.2$ where t is the age of the fish in years. Describe the ages of a group of Icelandic saithe that weigh up to 29 pounds.

$$w \leq 29 \longrightarrow 10.4t - 2.2 \leq 29$$

$$+2.2 \quad +2.2$$

$$\frac{10.4t}{10.4} \leq \frac{31.2}{10.4}$$

$$t \leq 3 \text{ years}$$

$F = \frac{9}{5}C + 32$

$-32 \quad -32$

$\frac{5}{9}(F-32) = \frac{5}{9}\left(\frac{9}{5}C\right)$

$C = \frac{5}{9}(F-32)$

Example 8

- You have added enough antifreeze to your car's cooling system to lower the freezing point to -35°C and raise the boiling point to 125°C. The coolant will remain a liquid as long as the temperature C (in degrees Celsius) satisfies the inequality $-35 < C < 125$. Write the inequality in degrees Fahrenheit.
- What is the relationship between C and F, solved for C?

$$C = \frac{5}{9}(F - 32)$$

Example 8

- For this problem begin with your inequality and your equation.
- -35<C<125

$$C = \frac{5}{9}(F - 32)$$

- Substitute in for C in the inequality so that the inequality is in terms of F.

$$-35 < \frac{5}{9}(F - 32) < 125$$

Example 8

- Solve. $-35 < \frac{5}{9}(F - 32) < 125$

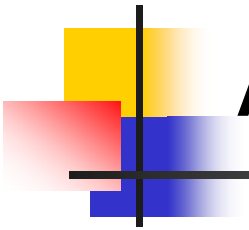
$-63 < F - 32 < 225$

$+32 \quad +32 \quad +32$

$-31 < F < 257$

Closure

- How does solving linear inequalities compare to solving linear equations?



Assignment

- Sec 1.6
  - 1, 2, 3-5, 9-17, 20, 21, 22-31 LC, 33, 34, 36, 41, 42, 46, 47, 49, 50, 53, 54, 56, 58