

Sec 1.2 Evaluate and Simplify Algebraic Expressions

Goals

- **Goal 1:** To evaluate algebraic expressions
- **Goal 2:** To simplify algebraic expressions by combining like terms

Definition

- A **numerical expression** consists of numbers, operations, and grouping symbols.
- We will now add **exponentiation, or power**, to the addition, subtraction, multiplication, & division we worked on last time.

Definition

- A **power, or exponentiation**, is repeated multiplication with the same factor, thus the term "raising to a power".

$$2^5 = \underbrace{2 \cdot 2 \cdot 2 \cdot 2 \cdot 2}_{5 \text{ factors of } 2}$$

Parts of a power

Exponent

2^5

Base

2^5 is a power

Example 1 Evaluate

a. $(-2)^6 = (-2)(-2)(-2)(-2)(-2)(-2) = 64$

b. $-3^4 = -3 \cdot 3 \cdot 3 \cdot 3 = -81$

Order of Operations

- What if you have powers, along with addition, subtraction, multiplication, & division?
 - Grouping symbols
 - Powers
 - Multiplication & division from left to right
 - Addition & subtraction from left to right

Order of Operations

- Mnemonic device: a way to help you remember something...
 - Roy G Biv
- Please Excuse My Dear Aunt Sally
 - Parentheses (& other grouping symbols)
 - Exponents (powers)
 - Multiplication & Division
 - Addition & Subtraction

Practice Order of Operations

~~P~~ ~~E~~ ~~MD~~ AS

$$-3 + 5(-2 + 4)^2 =$$

$$-3 + 5(2)^2$$

$$-3 + 5(4)$$

$$\uparrow \quad \uparrow$$

$$-3 + 20$$

$$17$$

Definitions

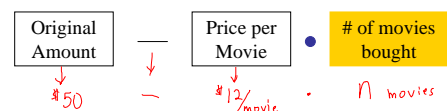
- A **variable** is a letter that is used to represent one or more numbers.
- A **value of a variable** is any number used to replace a variable.
- An **algebraic expression** is an expression involving variables.

Example 2

- Evaluate $-3x^2 - 5x + 7$ when $x = -3$.

Example 3 Real-Life Model

- You have \$50 and are buying some movies on videocassettes that cost \$12 each. Write an expression that shows how much money you have left after buying n movies. Evaluate the expression when $n = 2$ and $n = 3$.
- First, create a verbal model:



Example 3 Real-Life Model

- You have \$50 and are buying some movies on videocassettes that cost \$12 each. Write an expression that shows how much money you have left after buying n movies. Evaluate the expression when $n = 2$ and $n = 3$.
- Next, give labels to these items:
 - Original Amount = 50 (dollars)
 - Price per movie = 12 (dollars per movie)
 - Number of movies bought = n (movies)

Example 3 Real-Life Model

- You have \$50 and are buying some movies on videocassettes that cost \$12 each. Write an expression that shows how much money you have left after buying n movies. Evaluate the expression when $n = 2$ and $n = 3$.
- Next, create an algebraic model by replacing the verbal model with the labels:
 - $50 - 12n$

Example 3 Real-Life Model

- You have \$50 and are buying some movies on videocassettes that cost \$12 each. Write an expression that shows how much money you have left after buying n movies. Evaluate the expression when $n = 2$ and $n = 3$.
- Finally, answer the questions.

$$50 - 12n$$

$$\begin{array}{r} 50 - 12(2) \\ 50 - 24 \\ \hline 26 \end{array}$$

$$\begin{array}{r} 50 - 12(3) \\ 50 - 36 \\ \hline 14 \end{array}$$

Example 3 Real-Life Model

- Unit analysis:

$$\text{dollars} - 12n \left(\frac{\text{dollars}}{\text{movie}} \right) (\text{movies})$$

dollars - dollars = dollars

GUIDED PRACTICE for Examples 1, 2, and 3

Evaluate the expression.

1. $6^3 = 216$

2. $-2^5 = -32$

3. $(-2)^5 = -32$

4. $5x(x-2)$ when $x=6$

5. $3y^2 - 4y$ when $y=-2$

6. $(z+3)^3$ when $z=1$

$5(6)(6-2) = 5 \cdot 6 \cdot 4 = 120$

$3(-2)^2 - 4(-2) = 3 \cdot 4 + 8 = 20$

$(1+3)^3 = 4^3 = 64$

WHAT IF? In Example 3, find your profit if you sell 135 candles.

Definitions

- The **terms** of an expression are the parts that are added together.
 - There may be subtraction in there, but remember subtraction is the same as adding the opposite.
- A term has a variable part is called a **variable term**.

Definitions

- A term that has no variable part is called a **constant term**.
- When a term is the product of a number and a power of a variable, the **coefficient** is the number multiplied by the power.

Definitions

$$\begin{array}{cccc} 4x^3 & -2x^2 & +x & -6 \\ \text{VT} & \text{VT} & \text{VT} & \text{CT} \\ \text{Coeff 4} & -2 & 1 & \end{array}$$

- An expression is simplified if it contains no grouping symbols and all *like terms* are combined.
- Like terms** are terms that have the same variable parts.
- Constant terms** are also considered like terms.
- The distributive property allows you to combine like terms by adding coefficients.

Example 4

- a. $5x + 2x = (5+2)x = 7x$
- b. $4t^2 - t + t^2 = 5t^2 - t$
- c. $3(x+2) - 4(x-1) = 3x+6-4x+4 = -x+10$

Definitions

- Two algebraic expressions are equivalent expressions if they have the same value for all values of their variables.
- For instance, in part (a) of example for the expressions $5x + 2x$ and $7x$ are equivalent.
- A statement such as $5x + 2x = 7x$ that equates to equivalent equations is called an **identity**.

Example 5 Real-Life Model

- MUSIC: You want to buy either a CD or a cassette as a gift for each of 10 people. CDs cost \$12 each and cassettes cost \$7 each. Write an expression for the total amount you must spend. Then evaluate the expression when 4 of the people get CDs.
- First, create a verbal model:

Price per CD

•

of CDs

+

Price per cassette

•

of cassettes

$\$12/\text{CD}$

$n \text{ CDs}$

$\$7/\text{cass}$

$(10-n) \text{ cass}$

Example 5 Real-Life Model

- MUSIC: You want to buy either a CD or a cassette as a gift for each of 10 people. CDs cost \$12 each and cassettes cost \$7 each. Write an expression for the total amount you must spend. Then evaluate the expression when 4 of the people get CDs.
- Next, give labels to these items:
 - CD price = 12 (dollars per CD)
 - number of CDs = n (CDs)
 - cassette price = 7 (dollars per cassette)
 - number of cassettes = $10-n$ (cassettes)

Example 5 Real-Life Model

- MUSIC: You want to buy either a CD or a cassette as a gift for each of 10 people. CDs cost \$12 each and cassettes cost \$7 each. Write an expression for the total amount you must spend. Then evaluate the expression when 4 of the people get CDs.
- Next, create an algebraic model by replacing the verbal model with the labels:

- $12n + 7(10 - n)$

Example 5 Real-Life Model

- MUSIC: You want to buy either a CD or a cassette as a gift for each of 10 people. CDs cost \$12 each and cassettes cost \$7 each. Write an expression for the total amount you must spend. Then evaluate the expression when 4 of the people get CDs.

- Finally, answer the question:

$$\begin{array}{l} 12n + 7(10 - n) \\ 12(4) + 7(10 - 4) \\ 12(4) + 7(6) \\ 48 + 42 = \$90 \end{array} \quad \begin{array}{l} 12n + 70 - 7n \\ \cancel{12n} + 70 \\ 5n + 70 \\ 5(4) + 70 \\ 20 + 70 \\ \$90 \end{array}$$

Assignment

■ Sec 1.2

- 2, 3, 4-24 even, 28-30, 35, 41, 45-46, 52, 53, 56-58, 62