1.1 Apply Properties of Real Numbers

Before You performed operations with real numbers.

Now You will study properties of real numbers.

Why? So you can order elevations, as in Ex. 58.

Subsets of Real Numbers

- **Rational** : can be written as a fraction $\frac{\text{integer}}{\text{integer}}$
  as a decimal, the # will terminate or repeat (end)
  $\frac{2}{3}$ 1.38 $1\frac{19}{50}$ $\frac{1}{2}$ 5
- **Irrational** : Can't be written as a fraction
  As a decimal, will not terminate or repeat
  π $\sqrt{3}$ 1.010011000111...

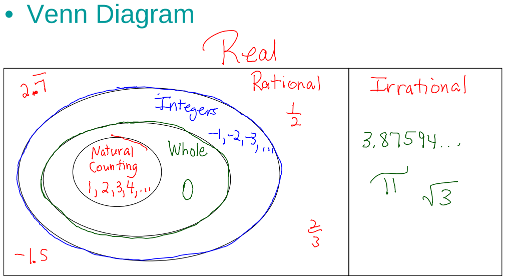
Subsets of Rational Numbers

- **Integers** ..., −3, −2, −1, 0, 1, 2, 3, 4, ...
- **Whole** : 0, 1, 2, 3, 4, ...
- **Natural** : (counting) : 1, 2, 3, 4, ...

KEY CONCEPT *For Your Notebook*

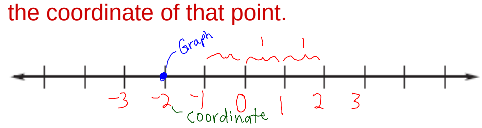
The Real Numbers

- Venn Diagram



KEY CONCEPT *For Your Notebook*

- Real numbers can be pictured as points on a line called a real number line, with the numbers increasing from left to right.
- The point labeled 0 (zero) is the origin.
- The point that corresponds to a real number is the graph of the number.
- The number that corresponds to a point is called the coordinate of that point.



EXAMPLE 1 Graph real numbers on a number line

$-2\frac{1}{3}$ $-2.\overline{3}$

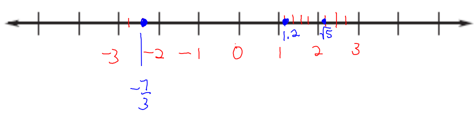
$-\frac{7}{3}$

≈ 2.24

$\sqrt{5}$

$1\frac{1}{5}$

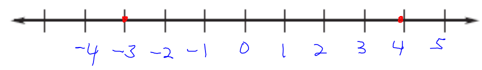
1.2



Ordering Numbers

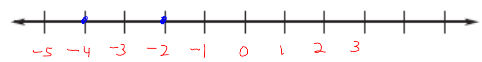
- -3 and 4

$-3 < 4$ $4 > -3$



- -2 and -4

$-2 < -4$



EXAMPLE 2 Ordering Elevations

- Here are the elevations of five locations in Imperial Valley, California.
  - Alamorio: –135 ft
  - Curlew: –93 ft.
  - Gieselmann Lake: –162 ft.
  - Moss: –100 ft
  - Orita: –92 ft
- Order from lowest to highest.

GL	Ala	Moss	Curlew	Orita
-162	-135	-100	-93	-92

✓ **GUIDED PRACTICE** for Examples 1 and 2

1. Graph the numbers -0.2, $\frac{7}{10}$, -1, $\sqrt{2}$, and -4 on a number line.

2. Which list shows the numbers in increasing order?

Ⓐ $-0.5, 1.5, -2, -0.75, \sqrt{7}$ Ⓑ $-0.5, -2, -0.75, 1.5, \sqrt{7}$

Ⓒ $-2, -0.75, -0.5, 1.5, \sqrt{7}$ Ⓓ $\sqrt{7}, 1.5, -0.5, -0.75, -2$

Properties of Real Numbers

Property	Addition	Multiplication
Closure	Real # + Real # = Real #	Real # X Real # = Real #
Commutative	$a+b = b+a$	$ab = ba$
Associative	$(a+b)+c = a+(b+c)$	$(ab)c = a(bc)$
* Identity	$a+0=a$ $0+a=a$	$b \cdot 1 = b$ $1 \cdot b = b$
* Inverse opposite reciprocal	$a + -a = 0$	$b \cdot \boxed{\frac{1}{b}} = 1$
Distributive	$a(b+c) = ab+ac$	$(a+b)c = ac+bc$

EXAMPLE 3 Identify properties of real numbers

- Identify the property shown:
  - a. 7 + 0 = 7
    - Identity
  - b. 4(3) = 3(4)
    - Commutative

Definitions

- The **opposite**, or *additive inverse*, of any number *a* is -*a*.
  - In algebra, subtraction is defined as adding the opposite.
- The **reciprocal**, or *multiplicative inverse*, of a number a is 1/a.
  - In algebra, division is defined as multiplying by the reciprocal.

Use the Definitions

- The difference of -8 and 7 is…

$\boxed{-8-7} \rightarrow -8 + -7 \; -15$

- The quotient of -12 and 1/4 is...

$-12 \div \frac{1}{4} \rightarrow -12 \cdot 4 = -48$

EXAMPLE 4 **Use properties and definitions of operations**

Use properties and definitions of operations to show that $\boxed{a + (2 - a)} = \boxed{2}$. Justify each step.

Solution

$a + (2 - a) = \boxed{a + [2 + (-a)]}$	Definition of subtraction
$= a + [(-a) + 2]$	Commutative property of addition
$= [a + (-a)] + 2$	Associative property of addition
$= 0 + 2$	Inverse property of addition
$= 2$	Identity property of addition

GUIDED PRACTICE for Examples 3 and 4

Identify the property that the statement illustrates.

3. $(2 \cdot 3) \cdot 9 = 2 \cdot (3 \cdot 9)$
4. $15 + 0 = 15$
5. $4(5 + 25) = 4(5) + 4(25)$
6. $1 \cdot 500 = 500$

Use properties and definitions of operations to show that the statement is true. *Justify* each step.

7. $\boxed{b \cdot (4 \div b)} = 4$ when $b \neq 0$

$b \cdot (4 \cdot \frac{1}{b}) \rightarrow$ def. of division

$b \cdot (\frac{1}{b} \cdot 4) \rightarrow$ commutative

$(b \cdot \frac{1}{b}) \cdot 4 \rightarrow$ associative

$1 \cdot 4 \rightarrow$ inverse

$4 \rightarrow$ identity

8. $\boxed{3x + (6 + 4x)} = \boxed{7x + 6}$

$3x + (4x + 6) \rightarrow$ comm.

$\boxed{(3x + 4x)} \boxed{+6} \rightarrow$ assoc.

$(3+4)x + 6 \rightarrow$ distrib.

$7x + 6 \rightarrow$ addition

Unit Analysis

- When you add, subtract, multiply, and divide in real life, you should use *unit analysis* to make sure that the units in your answer makes sense.

EXAMPLE 5 **Use unit analysis with operations**

You are going on a 497 mile trip. So far you have traveled 315 miles. How much farther?

497 miles - 315 miles = 182 miles

You traveled for 3 hours at a speed of 65 miles per hour. How far did you go?

$(3 \cancel{\text{hours}})\left(\frac{65 \text{ miles}}{1 \cancel{\text{hour}}}\right) =$ 195 miles

EXAMPLE 5 Use unit analysis with operations

You work 4 hours and earn $36. What is your earning rate? $\frac{\$}{hr}$

$$\frac{36 \text{ dollars}}{4 \text{ hours}} = \frac{\$9}{hr}$$

Use unit analysis to determine what the following calculation does:

$$\left(\frac{88 \cancel{\text{ feet}}}{1 \cancel{\text{ second}}}\right)\left(\frac{3600 \cancel{\text{ seconds}}}{1 \text{ hour}}\right)\left(\frac{1 \text{ mile}}{5280 \cancel{\text{ feet}}}\right) = \boxed{\frac{316{,}800 \text{ mi}}{5280 \text{ hr}}}$$

$60 \text{ mi}/\text{hr}$

EXAMPLE 6 Use unit analysis with conversions

DRIVING DISTANCE The distance from Montpelier, Vermont, to Montreal, Canada, is about 132 miles. The distance from Montreal to Quebec City is about 253 kilometers.

a. Convert the distance from Montpelier to Montreal to kilometers.

b. Convert the distance from Montreal to Quebec City to miles.

a) $\frac{132 \cancel{mi}}{1} \cdot \frac{1.61 \text{ km}}{1 \cancel{mi}} = 212.52 \text{ km}$

b) $\frac{253 \cancel{km}}{1} \cdot \frac{1 \text{ mi}}{1.61 \cancel{km}} = \frac{253}{1.61} \text{ mi} = 157.14 \text{ mi}$

✓ GUIDED PRACTICE for Examples 5 and 6

Solve the problem. Use unit analysis to check your work.

9. You work 6 hours and earn $69. What is your earning rate?
10. How long does it take to travel 180 miles at 40 miles per hour?
11. You drive 60 kilometers per hour. What is your speed in miles per hour?

Perform the indicated conversion.

12. 150 yards to feet
13. 4 gallons to pints
14. 16 years to seconds

to get decimal instead of fraction: ctrl $\overset{\approx}{\boxed{\text{enter}}}$

1.1 EXERCISES

HOMEWORK KEY

○ = **WORKED-OUT SOLUTIONS** on p. WS1 for Exs. 21, 31, and 59

★ = **STANDARDIZED TEST PRACTICE** Exs. 2, 9, 10, 23, 24, 60, and 61

- ***2, 3, 7, 9-16 all, 17-23 odd, 26-30 even, 31, 34, 35, 42, 48, 50, 58, 60, 62, 70, 71, 73, 75, 79***
- ***There will be a homework quiz next time. You may use this homework on your quiz.***
- ***You may add notes to your homework, but you may not use the notes sheets from class when you take the quiz.***
- ***You may come in any time before class and check your answers.***